A SYSTEMATIC APPROACH TO ANOMALOUS PHENOMENA AT HIGH ENERGIES

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In one-dimensional case the search for presence of the anomalous phenomena in multiplicity distributions is usually performed in frame of the horizontal, vertical and mixed types of the analysis. We show that if the data involve a d-dimensional phase space, there exists a convenient procedure, generalizing the one-dimensional approach, which allows to introduce more non-trivial types of the analysis; we formulate them in some detail in one-, two-, and partially also in three-dimensions.

1 Introduction

Search for anomalous phenomena at high energies is usually performed in terms of a convenient kind of statistical moments, the information about the presence of those phenomena being encoded in appropriate sort of the scaling indices. Quite often, the one-dimensional cases are considered[1] and usually three types of the analysis are applied there[2], namely, the *horizontal* (H), *vertical* (V), and the *mixed* (HV) ones. However, there is still missed a systematic approach to higher dimensions as well as a deeper rooted physical theory or at least a more or less reliable dynamical model for the scaling indices characterizing those phenomena.

Let us add that in the cases considered in the present contribution, the appearance of the non-statistical, anomalous fluctuations, induces the need to take into account the non-continuous statistical distributions.

In principle, the notion "anomalous" means (special, see below) "non-statistical" while "statistical" denotes "pure Poissonian".

When investigating the presence and properties of the anomalous phenomena in high energy physics, the factorial moments of rank q are usually applied, their normalization being such that for the pure Poissonian distribution they are equal to unity (for q= 1, 2, 3, ...). This means that their deviation of unity (at least for some values of the rank q) signifies the presence of the non-statistical behavior in the corresponding data.

In the present contribution a sample of E events is investigated, the counting index of the events being denoted by e, i.e. e=1, 2, 3, ..., E. The number of (charged) particles observed in a given region of the phase-space under consideration is denoted by n. In what follows also the factorial multinomial, $\mathcal{F}$, introduced by

$$\mathcal{F} \; = \; \mathcal{F}(n;q) \equiv \; n(n-1)\dots(n-q+1) \; , \tag{1}$$

is applied.

2 One-dimensional case

We assume that the dependence of the multiplicity distribution of (charged, produced) secondaries on one variable (say, the pseudorapidity η of every particle under consideration) is known from the observed collisions.

Let the multiplicity data be known in a pseudorapidity window $\Delta\eta$, and, eventually, this window be partitioned into M equal size bins, $\delta\eta, \delta\eta = (\Delta\eta)/M$; the index counting the bins is denoted by m, i.e. $m = 1, 2, 3, \dots, M$. It is understood that the number of particles in every elementary cell specified by two numbers, namely, (me), i.e. n_{me}, is deduced from the experimental output.

Usually it is said that the anomalous phenomenon (intermittency) is present if the experimental data exhibit (with increasing number of bins, M) a linear dependence between the logarithm of the factorial moments and logarithm of the number of bins, M, i.e., if the factorial moments reveal a singularity of the form M^{a_q}, a_q being non-vanishing quantities; they are called slopes or scaling indices[3]. It is worthwile to introduce also the following notations: to emphasize the **I**ndividual, **I**ndependent averaging of the expression entering the numerator *or* denominator of the corresponding moment,

$$\mathbf{I}_m \equiv \frac{1}{M} \sum_{m=1}^{M} \; , \quad \mathbf{I}_e \equiv \frac{1}{E} \sum_{e=1}^{E} \; ,$$

and, to emphasize the averaging over **B**oth, the numerator *as well as* the denominator,

$$\mathbf{B}_m \; \equiv \; \frac{1}{M} \sum_{m=1}^{M} \; , \quad \mathbf{B}_e \equiv \; \frac{1}{E} \sum_{e=1}^{E} \; .$$

Now, application of the *horizontal* type of the analysis means that the factorial moments in the $(me)-$th cell $[F(q;M)]^{(H)}_{me}$, in the $e-$th event $[F(q;M)]^{(H)}_{e}$, and in the whole sample $[F(q;M)]^{(H)}$, are introduced, respectively, by

$$[F(q;M)]^{(H)}_{me} \; = \; \frac{\mathcal{F}(n = n_{me};q)}{[\mathbf{I}_{m'} n_{m'e}]^q} \tag{2}$$

and

$$[F(q;M)]_e^{(H)} = \mathbf{I}_m [F(q;M)]_{me}^{(H)}$$
$$= \frac{\mathbf{I}_m \mathcal{F}(n = n_{me}; q)}{[\mathbf{I}_{m'} n_{m'e}]^q} , \quad (3)$$

$$[F(q;M)]^{(H)} = \mathbf{B}_e [F(q;M)]_e^{(H)} \quad (4)$$

where $\mathcal{F}$ is given by Eq. (1). In this case the scaling properties are investigated by means of the relation,

$$[F(q;M)]^{(H)} \propto f_q^{(H)} M^{a_q^{(H)}} . \quad (5)$$

The *vertical* type of the analysis is specified by the following form of the factorial moments,

$$[F(q;M)]_{me}^{(V)} = \frac{\mathcal{F}(n = n_{me}; q)}{[\mathbf{I}_{e'} n_{me'}]^q} \quad (6)$$

and

$$[F(q;M)]_m^{(V)} = \mathbf{I}_e [F(q;M)]_{me}^{(V)} \quad (7)$$

$$[F(q;M)]^{(V)} = \mathbf{B}_m [F(q;M)]_m^{(V)}$$
$$= \mathbf{B}_m \mathbf{I}_e [F(q;M)]_{me}^{(V)} \quad (8)$$

where $\mathcal{F}$ is still given by (1). Now, the scaling property is formulated in the following way,

$$[F(q;M)]^{(V)} \propto f_q^{(V)} M^{a_q^{(V)}} . \quad (9)$$

The *mixed* type of the analysis is characterized by the following expressions,

$$[F(q;M)]_{me}^{(HV)} = \frac{\mathcal{F}(n = n_{me}; q)}{[\mathbf{I}_{m'} \mathbf{I}_{e'} n_{m'e'}]^q} \quad (10)$$

and

$$[F(q;M)]^{(HV)} = \mathbf{I}_m \mathbf{I}_e [F(q;M)]_{me}^{(HV)} \propto f_q^{(HV)} M^{a_q^{(HV)}} . \quad (11)$$

We conclude this section by the observation that the expressions

$$\mathbf{B}_e \mathbf{I}_m , \quad \mathbf{B}_m \mathbf{I}_e , \quad \mathbf{I}_m \mathbf{I}_e$$

represent, respectively, a shorthand notation of the three types of the analysis mentioned above. It is perhaps worth to mention that in case of the silicon (at 14.6 A GeV/c) colliding with the emulsion nuclei (the BNL data), those three types of the analysis lead[4] to similar values of the slopes (scaling indices) a_q, with $q = 2, 3, 4, 5$.

3 Two-dimensional case

Let us investigate the presence of non-statistical fluctuations in the (charged) multiplicity distributions depending on two variables (like, e.g., the pseudo-rapidity and azimuthal angle). We present shortly the way which allows to formulate the relations appropriate to this case.

First of all, we introduce the partitioning of the windows where both quantities mentioned above were measured. The corresponding bins are counted by the indices m_1 and m_2 where

$$1 \ \leq \ m_1 \ \leq \ M_1 \quad \text{and} \quad 1 \ \leq \ m_2 \ \leq M_2 \ .$$

The number of charged particles, n, observed in the (m_1, m_2, e)-th elementary cell is denoted by

$$n \ = \ n_{m_1,m_2,e} \ . \tag{12}$$

In this case the following notation is useful,

$$\mathbf{I}_{m_1} \ \equiv \ \mathbf{I}_1 \ , \quad \mathbf{I}_{m_2} \ \equiv \mathbf{I}_2 \ ; \quad \mathbf{B}_{m_1} \ \equiv \mathbf{B}_1 \ , \quad \mathbf{B}_{m_2} \ \equiv \ \mathbf{B}_2$$

and $\mathbf{I}_e$, $\mathbf{B}_e$ represent the same averaging as in the preceding Section.

There are seven non-trivial types of the analysis; they can be represented in the following form,

$$\begin{array}{ll}
[\mathbf{2dim;1}] \ : \ \mathbf{B}_e\mathbf{B}_2\mathbf{I}_1, & [\mathbf{2dim;4}] \ : \ \mathbf{B}_e\mathbf{I}_2\mathbf{I}_1 \ , \\
[\mathbf{2dim;2}] \ : \ \mathbf{B}_e\mathbf{B}_1\mathbf{I}_2, & [\mathbf{2dim;5}] \ : \ \mathbf{B}_2\mathbf{I}_e\mathbf{I}_1 \ , \\
[\mathbf{2dim;3}] \ : \ \mathbf{B}_2\mathbf{B}_1\mathbf{I}_e, & [\mathbf{2dim;6}] \ : \ \mathbf{B}_1\mathbf{I}_e\mathbf{I}_2 \ , \\
\multicolumn{2}{c}{[\mathbf{2dim;7}] \ : \ \mathbf{I}_e\mathbf{I}_2\mathbf{I}_1 \ .}
\end{array} \tag{13}$$

For instance, in the first case, **[2dim; 1]**,

$$\begin{aligned}
F_{m_1,m_2,e}(q) &= \frac{\mathcal{F}(n;q)}{[\mathbf{I}_1 n]^q} \ , \\
F_{m_2,e}(q) &= \mathbf{I}_1 F_{m_1,m_2,e}(q) = \frac{\mathbf{I}_1\mathcal{F}(n;q)}{[\mathbf{I}_1 n]^q} \ , \\
F(q) &= \mathbf{B}_e\mathbf{B}_2 \frac{\mathbf{I}_1\mathcal{F}(n;q)}{[\mathbf{I}_1 n]^q} \propto f_q(M_1 M_2)^{a_q}
\end{aligned} \tag{14}$$

where the multiplicity n of the elementary cells is given by Eq. (12).

4 Three-dimensional case

If the question is to be answered whether the non-statistical fluctuations are present in multiplicity data depending simultaneously on three variables (like e.g. the pseudorapidity, azimuthal angle and transverse momentum), first of all the number of (charged) particles in every elementary cell should be specified. To this end, the windows in all three variables are partitioned thereby involving the relations, $1 \leq m_j \leq M_j$, with $j = 1, 2, 3$. In this case, the multiplicity n of the elementary cell is specified by

$$n \quad = \quad n_{m_1,m_2,m_3,e} \quad . \tag{15}$$

Introducing the notation analogous to that one applied in the preceding cases, namely

$$\mathbf{I}_{m_j} \equiv \mathbf{I}_j \quad \text{and} \quad \mathbf{B}_{m_j} \equiv \mathbf{B}_j \quad j = 1, 2, 3,$$

we observe that in the present case the following fifteen possible types of the analysis can be introduced,

$$\begin{array}{lcl}
[\mathbf{3dim;1}] : \mathbf{B}_e\mathbf{B}_3\mathbf{B}_2\mathbf{I}_1 & , & [\mathbf{3dim;3}] : \mathbf{B}_e\mathbf{B}_2\mathbf{B}_1\mathbf{I}_3 \\
[\mathbf{3dim;2}] : \mathbf{B}_e\mathbf{B}_3\mathbf{B}_1\mathbf{I}_2 & , & [\mathbf{3dim;4}] : \mathbf{B}_3\mathbf{B}_2\mathbf{B}_1\mathbf{I}_e \\
 & & \\
[\mathbf{3dim;5}] : \mathbf{B}_e\mathbf{B}_3\mathbf{I}_2\mathbf{I}_1 & , & [\mathbf{3dim;8}] : \mathbf{B}_e\mathbf{B}_1\mathbf{I}_3\mathbf{I}_2 \\
[\mathbf{3dim;6}] : \mathbf{B}_e\mathbf{B}_2\mathbf{I}_3\mathbf{I}_1 & , & [\mathbf{3dim;9}] : \mathbf{B}_3\mathbf{B}_1\mathbf{I}_e\mathbf{I}_2 \\
[\mathbf{3dim;7}] : \mathbf{B}_3\mathbf{B}_2\mathbf{I}_e\mathbf{I}_1 & , & [\mathbf{3dim;10}] : \mathbf{B}_2\mathbf{B}_1\mathbf{I}_e\mathbf{I}_3 \\
 & & \\
[\mathbf{3dim;11}] : \mathbf{B}_e\mathbf{I}_3\mathbf{I}_2\mathbf{I}_1 & , & [\mathbf{3dim;13}] : \mathbf{B}_2\mathbf{I}_e\mathbf{I}_3\mathbf{I}_1 \\
[\mathbf{3dim;12}] : \mathbf{B}_3\mathbf{I}_e\mathbf{I}_2\mathbf{I}_1 & , & [\mathbf{3dim;14}] : \mathbf{B}_1\mathbf{I}_e\mathbf{I}_3\mathbf{I}_2 \\
 & & \\
\multicolumn{3}{c}{[\mathbf{3dim;15}] : \mathbf{I}_e\mathbf{I}_3\mathbf{I}_2\mathbf{I}_1 \ .}
\end{array} \tag{16}$$

In Eq. (16), the $\mathbf{B}_e$ and $\mathbf{I}_e$ have the same meaning as in the preceding Sections. For instance, the first type of the analysis mentioned in Eq. (16) involves the factorial moments in the (m_1, m_2, m_3, e)-th cell,

$$F_{m_1,m_2,m_3,e}(q) = \frac{\mathcal{F}(n;q)}{[\mathbf{I}_1 n]^q} \tag{17}$$

as well as the full form of that moment together with the corresponding scaling condition,

$$F(q) = \mathbf{B}_e\mathbf{B}_3\mathbf{B}_2\mathbf{I}_1 F_{m_1,m_2,m_3,e}(q) \quad \propto \quad f_q(M_1M_2M_3)^{a_q} \tag{18}$$

where the multiplicity n is given by Eq. (15).

Let us add that every type of the analysis is characterized by its own set of the scaling indices a_q and the corresponding form of all kinds of the associated moments (like the frequency, correlation, dispersion, etc. moments); some more details in[5].

5 Dispersion moments

It is well known that several multiplicity distributions observed in the past, were satisfactorily described by continuous statistical distributions inherent in different modelling ideas. And quite often the associated moments, especially the corresponding dispersion moments, represented a very apt tool which helped to discriminate between more and less appropriate models. Now, it can be expected that especially at higher energies (available today as well as in the future) the possible presence of the intermittency should be taken into account. In this case, application of the continuous distributions is already not sufficiently adequate, compare in particular e.g. Fig. 1 in[6]; that distribution, in[6], representing one event, is induced by non-self-similar processes and clearly reveals scaling property[7] (also other authors deal with the individual events, for instance[8]). Application of the corresponding associated statistical moments (like e.g. the appropriately introduced dispersion moments), might play again a decisive role in the procedure leading to the discern between the more or less reliable fractal model approaches. Moreover, the dispersion moments involve also a piece of information about the presence of the empty bins.

In particular, the first type of the analysis mentioned in the present paper, **[1dim; 1]**, involves (let us call them "modified") dispersion moments $\tilde{D}$ defined by the following way

$$[\tilde{D}^q_q]_{me} = \left(\frac{n_{me}}{\mathbf{I}_{m'} n_{m'e}} - 1 \right)^q \tag{19}$$

and $\tilde{D}^q_q = \mathbf{B}_e \mathbf{I}_m [\tilde{D}^q_q]_{me}$. On the other hand, the "standard" dispersion moments D are introduced by

$$[D^q_q]_{me} \ = \ (n_{me} - \mathbf{I}_{m'} n_{m'e})^q \ , \tag{20}$$

i.e. their relation with the modified $\tilde{D}$-dispersion moments acquires the form

$$[\tilde{D}^q_q]_{me} \ = \ \frac{1}{[\mathbf{I}_{m'} n_{m'e}]^q} [D^q_q]_{me} \tag{21}$$

and

$$\tilde{D}^q_q \ = \ \mathbf{B}_e \frac{\mathbf{I}_m [D^q_q]_{me}}{[\mathbf{I}_{m'} n_{m'e}]^q} \ = \ \mathbf{B}_e \mathbf{I}_m [\tilde{D}^q_q]_{me} \ . \tag{22}$$

Moreover, in the case **[2dim; 1]**:

$$[\tilde{D}_q^q]_{m_1,m_2,e} = \left(\frac{n_{m_1,m_2,e}}{\mathbf{I}_{m_1'} n_{m_1',m_2,e}} - 1 \right)^q \quad , \quad \tilde{D}_q^q = \mathbf{B}_e \mathbf{B}_{m_2} \mathbf{I}_{m_1} [\tilde{D}_q^q]_{m_1,m_2,e} \ ; \tag{23}$$

and **[3dim; 1]**:

$$[\tilde{D}_q^q]_{m_1,m_2,m_3,e} = \left(\frac{n_{m_1,m_2,m_3,e}}{\mathbf{I}_{m_1'} n_{m_1',m_2,m_3,e}} - 1 \right)^q \ ,$$

$$\tilde{D}_q^q = \mathbf{B}_e \mathbf{B}_{m_3} \mathbf{B}_{m_2} \mathbf{I}_{m_1} \left(\frac{n_{m_1,m_2,m_3,e}}{\mathbf{I}_{m_1'} n_{m_1',m_2,m_3,e}} - 1 \right)^q \ . \tag{24}$$

We note that the multifractal analysis (of the type **[1dim; 1]**) of data coming from the collisions of gold (at 10.6 A GeV/c) with emulsion nuclei[9] leads to the conclusion that the lower rank dispersion moments (essentially of the form of Eq. (19)) reveal scaling property with sufficient accuracy[10].

Analogous procedure can be applied also in higher dimensional cases.

6 Conclusions

In the present contribution, investigating presence of the anomalous fluctuations, a natural and systematic extension of the one-dimensional approach to arbitrary, say, d-dimensional case is proposed; it involves

$$\sum_{j=0}^{d} \binom{d+1}{j} \ = \ 2^{d+1} - 1$$

non-trivial types of the analysis. The aim is to apply such a systematic approach to a same set of the data and to look for the regularities embedded in the scaling indices which belong to the same branch of the analyses. Hopefully, the regularities which would be observed there might lead at least to the construction of the corresponding model equations.

Especially, it is expected that a systematic application of the non-trivial types of the analysis to several samples of events will allow (i) to recognize which type of the analysis is the most suitable one for detecting the presence of the concrete type of the anomalous fluctuations, (ii) to introduce an appropriate cathegorization of the anomalous phenomena, (iii) to ascribe a concrete numerical value to the parameters which enter theoretical expressions applied to the description of the experimental data, and, eventually, (iv) to formulate a sufficiently reliable (and still missing) theory (or at least model) of the anomalous phenomena.

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